

# On General Optimisation of a Stochastic Toll-Booth System Proposed in High Traffic Density Conditions

Rohan Shah

UG Student

Civil Engineering Dept.

Dr. Prabhat Shrivastava

Professor & Head

Civil Engineering Dept.



Sardar Patel College of Engineering  
University of Mumbai  
Andheri (West), Mumbai-58

*Conference on Urban Mobility-Challenges, Solutions and Prospects  
(UMCSP)  
July 13-14, 2012  
IIT Madras*

# Brief Contents

- ▣ Introduction- Toll Booth Systems and Stochasticity, Site of Study
- ▣ Methodology – Multi-phased
- ▣ Problem Formulation
- ▣ Results and Discussion

# Introduction-Toll Booth Systems

- ▣ Randomised processes of arrival and serving
- ▣ Relation to Queueing Models
- ▣ High dissatisfaction levels among users found today

# Site of Study

- ▣ The Versova –Bandra Sea – Link , Mumbai
- ▣ Proposed under MSRDC
- ▣ Connection with existing Bandra-Worli Sea Link
- ▣ Aim – Alleviation of suburban congestion on the parallel networks
- ▣ Connector-based entries



Bandra

Worli

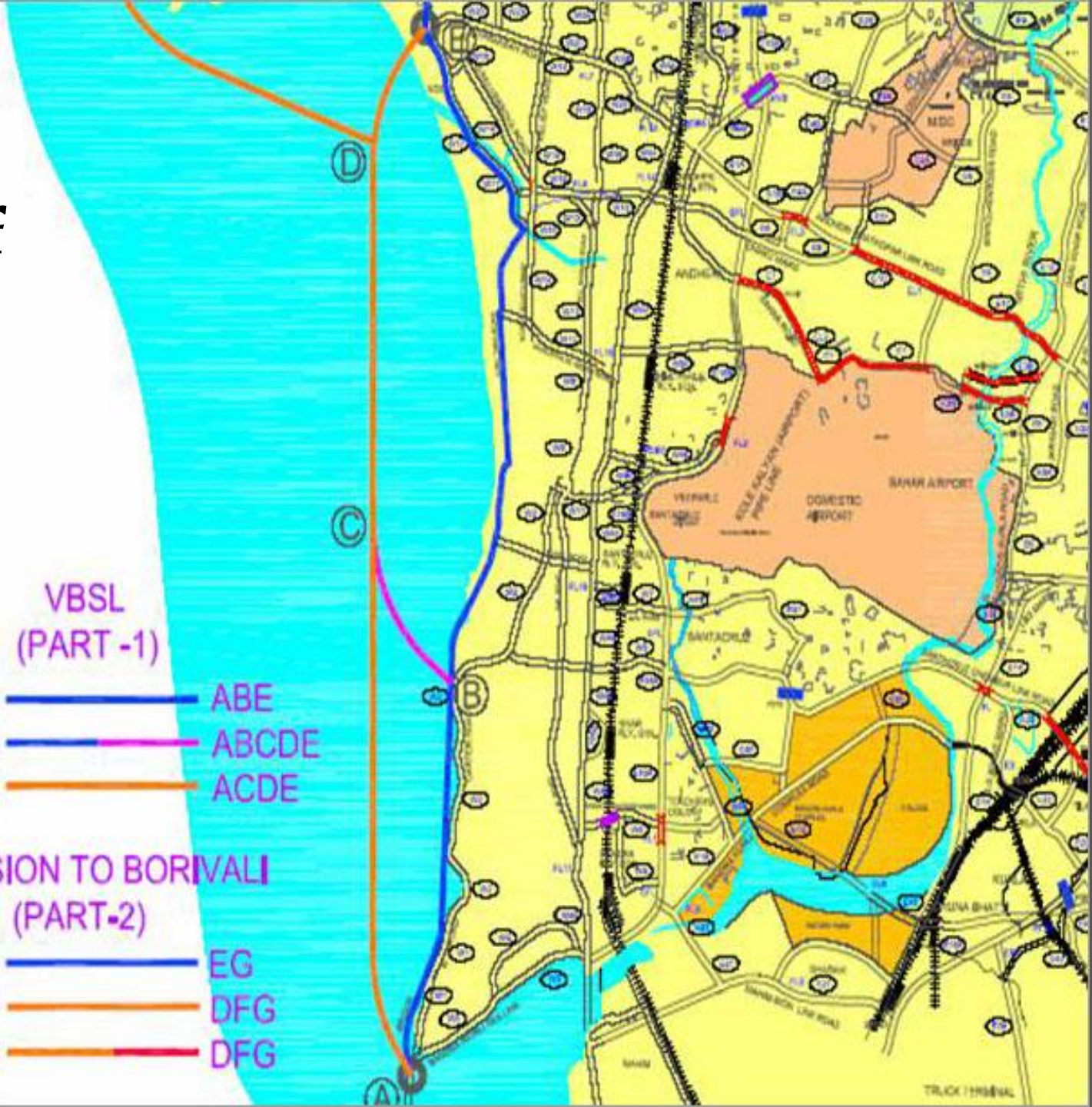
Bandra

Worli



# Site of Study

- VBSL  
(PART -1)
- Option 1 — ABE
  - Option 2 — ABCDE
  - Option 3 — ACDE
- EXTENSION TO BORIVALI  
(PART-2)
- Option 1 — EG
  - Option 2 — DFG
  - Option 3 — DFG



# Methodology : Step 1

- ▣ Identification of the Queueing Model
- ▣ conventional process - M/M/1 models
- ▣ Not fully realistic in high traffic density conditions due to high variation of parameters from actual
- ▣ queue length and wait time get overestimated
- ▣ We use the M/G/1 model
- ▣ General Distribution of Service Times

# Methodology : Step-2

- ▣ Traffic Assignment Modelling to Estimate the Expected Demand
- ▣ Lack of on-field volumes
- ▣ Cube 5.0 used for demand forecasting
- ▣ 'HIGHWAY' application used
- ▣ Input 1 - The network file of Mumbai region (completed with attributes of each existing network including counts, speed-delays, tolls)



# 'HIGHWAY' Application



# Methodology : Step 3- M/G/1 Condition Parameters

- ▣ Assumption - Mean arrival rate per lane type is uniformly distributed across each lane
- ▣ (total number of lanes)  $N$ -large enough to efficiently process transactions

$$\sum_{i=1}^K n_i = N$$

# M/G/1 parameters

- ▣  $\mu_i$  = *the mean service rate*
- ▣  $\lambda_i$  = *the mean arrival rate*

$$\frac{\lambda}{n} < \mu_i$$

Condition to prevent bulking and avoid  
formation of infinite queues

# M/G/1 parameters

- ▣  $L_i =$  the mean queue length, i.e., the mean number of vehicles waiting in the queue in type  $i$  lane
- ▣ The Pollaczek–Khintchine (P-K) Formula for M/G/1 models –

$$L_i = \frac{\lambda(\mu^2\sigma^2 + 1)}{2\mu(\mu - \frac{\lambda}{n})}$$

$\sigma_i =$  standard deviation of service time for a type  $i$  lane

- ▣ But,  $L_i$  not always a good criterion for performance comparison since services may differ over different lanes
- ▣ Mean waiting time in the queue impacts more
- ▣ Directly affects perception of service quality at the toll plaza by the motorists



- ▣ Hence, using Little's Law

$$W_i = \frac{L_i}{\lambda_i}$$

- ▣  $W_i =$  mean waiting time in queue for type  $i$  lane
- ▣ Then-

$$W = \frac{\lambda(\mu^2\sigma^2 + 1)}{2n\mu(\mu - \frac{\lambda}{n})}$$

# Methodology : Step 4- Optimisation Scenario

- ▣ General toll operations involve two major costs – operating costs and user-waiting costs
- ▣ different magnitudes associated with peak and off-peak times
- ▣ aims of operation not be the same between peak and off-peak times.
- ▣ **Peak hours** - exploitation of plus-points more important than savings in operating costs
- ▣ Hence higher weights for user-waiting costs.
- ▣ **Off-peak hours** - congestion low
- ▣ Service standards well-satisfied even with partial capacity operation. Thus, higher weights are given to operating costs

# Optimisation Scenario-Integer Programming

minimise

$$Z = \sum_{i=1}^K c_i n_i + d \sum_{i=1}^K n_i^2$$

( $c_i$  = rate of the operating cost of a type  $i$  lane

$d$  = rate of the value of time to the driver waiting in queue)

subject to

$$1) \quad \frac{\lambda_i}{n_i} < \mu_i \forall i$$

$$2) \quad \sum_{i=1}^K n_i = N$$

(peak hours)

$$3) \sum_{i=1}^K n_i \leq N \quad (\text{non peak hours})$$

$$4) W = \frac{\lambda(\mu^2\sigma^2 + 1)}{2\mu(\mu - \frac{\lambda}{n})}$$

$$5) n_i = \text{integer} \quad \forall i$$

# Optimisation Formulation

- ▣ Objective function  $Z$  represents the sum of the total operating costs for all lanes used and the total user waiting costs incurred by all drivers in the queues per unit time
- ▣ To find the optimal lane configuration to minimize the total costs  $Z$  by satisfying the constraints.



# Optimisation Solution

- ▣ Mumbai Scenario – no Automatic/Electronic Toll implemented before
- ▣ Hence solved for the manual lane scenario
- ▣ Waiting Time in queue = 40 sec
- ▣ Serving Time for Manual Server = 10-15 sec

# Optimisation Solution

- ▣ Solution yields effective lane configuration
- ▣ Total six number of lanes required
- ▣ At some points, objective achieved, but service rate becomes less than arrival rate
- ▣ But at six, service rate satisfactory
- ▣ System remains stable with finite queues

# Optimisation Solution

- ▣ Peak Hours - 08.00-11.00 hrs morning and 17.00-21.00 hrs evening in (Mumbai suburban scenario)  
– Full Capacity Operation Required
- ▣ Partial Capacity Operation also permitted
- ▣ Non-peak hours -(remaining times of day), two of the lanes maybe closed
- ▣ save manpower and operation-cost resources, without much affecting serving rates

# Scope for Automated Tolling

- ▣ In principle, service improvement easily by providing additional booths
- ▣ In practise, space and cost restrictions disallow that
- ▣ Multiple Lane Configuration may hold the key
- ▣ Automatic Coin Machine (ACM) Lane, even the Electronic Toll Collection (ETC) to expedite the process.
- ▣ ETC is too expensive, the ACM can be implemented (enabled with currency note acceptance as high value coins are just beginning to be available in India)
- ▣ service rate much faster compared to manual (serving time of as low as 5-7 seconds due to the elimination of human factors)

# Discussion

- ▣ configuration of (5,1)≡(Manual, ACM)
- ▣ a service improvement over the all-manual configuration
- ▣ Other benefit – encouragement to Mumbai drivers to start using it
- ▣ a positive step towards initiation of ETC lanes in future, like in USA, Germany
- ▣ Currently only one ETC-enabled system in India at Gurgaon



# Discussion (..contd)

- ▣ Mumbai City desperately needs enhanced transportation solutions
- ▣ General driver- discontentment regarding transportation systems is growing each day
- ▣ Vehicle fleet growing each day
- ▣ This configuration (including automatic) first of its kind (even recent BWSL does not have)
- ▣ Possible future upgradation to ETC
- ▣ This work promotes LOS improvement right at planning stages of VBSL

# Future Scope

- ▣ Feasibility evaluation of ETC-lanes for the VBSL
- ▣ Evaluating effective strategies to encourage current manual lane users to adopt automatic lanes
- ▣ Considering growth of traffic volume which may occur every year- working on a Dynamic Lane Configuration for it

# References

- Al-Deek, H.M., Mohamed, A.A., Radwan, A.E., 1997. Operational benefits of electronic toll collection: case study. *Journal of Transportation Engineering*, 123.
- Boronico, J., Siegel, P.H., 1998. Capacity planning for toll roadways incorporating consumer wait time costs. *Transportation Research Part A* 32, 297–310.
- Heyman, D., 1968. Optimal operating policies for M/G/1 queueing systems. *Journal of the Operational Research Society of America* 16, 362–382.
- Klodzinski, J., Al-Deek, H.M., 2002. Proposed level of service methodology for a toll plaza. *Transportation Research Record* 1802, 86–96.
- Heidemann, D., Wegmann, H., 1997. Queueing at unsignalized intersections. *Transportation Research B* 31, 239-263.
- Kim S (2009), The toll plaza optimization problem: Design, operations, and strategies. *Transportation Research Part E* 45 (2009) 125–137
- Heidemann, D., 1994. Queue length and delay distributions at traffic signals. *Transportation Research B* 28, 377-389.
- Murat A., Keith K.,, 2009. Development of Methodology for Toll Plaza Delay Estimation for Use in Travel Demand Model Postprocessor. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2133, Washington, D.C., 2009, 1–10.
- Bell M. G. H. (1994) Stochastic user equilibrium assignment in networks with queues. *Transportation Research B* 29, 125-137.
- Taha H.A. *Operations Research-An Introduction*.8<sup>th</sup> Edition. Pearson, Inc.2007, NJ-USA

**THANK YOU**  
**-RESP. PROFESSORS**  
**- FELLOW STUDENTS**  
**-ORGANISERS OF UMCSP, IIT MADRAS**

